Performance of an adaptive beamforming noise reduction scheme for hearing aid applications. I. Prediction of the signal-to-noise-ratio improvement

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Adaptive beamformers have been proposed as noise reduction schemes for conventional hearing aids and cochlear implants. A method to predict the amount of noise reduction that can be achieved by a two-microphone adaptive beamformer is presented. The prediction is based on a model of the acoustic environment in which the presence of one acoustic target-signal source and one acoustic noise source in a reverberant enclosure is assumed. The acoustic field is sampled using two omnidirectional microphones mounted close to the ears of a user. The model takes eleven different parameters into account, including reverberation time and size of the room, directionality of the acoustic sources, and design parameters of the beamformer itself, including length of the adaptive filter and delay in the target signal path. An approximation to predict the achievable signal-to-noise improvement based on the model is presented. Potential applications as well as limitations of the proposed prediction method are discussed and a FORTRAN subroutine to predict the achievable signal-to-noise improvement is provided. Experimental verification of the predictions is provided in a companion paper [J. Acoust. Soc. Am. 109, 1134 (2001)]. © 2001 Acoustical Society of America. [DOI: 10.1121/1.1338557]

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LIST OF STMBOLS		$g_{\mathrm{nR},i}$	<i>i</i> th coefficient of filter G_{nR}
A,B,C,D	models of impulse responses between acous-	$g_{\mathrm{nL},i}$	<i>i</i> th coefficient of filter G_{nL}
	tic sources and input of adaptive filter (cf. Σ^{2}	h	noise reduction of the adaptive filter, defined
	Fig. 2)	1	as $E\{a^-\} = E\{\epsilon^-\}$
a _i	ith coefficient of filter A	ĸ	sample index
b_i	ith coefficient of filter B	l _n	distance between noise source and center of
С	sound speed, m/s		listener's head, m
d	sum of both microphone signals, delayed by Δ samples	l_s	distance between target signal source and center of listener's head, m
d'	sum of both microphone signals	п	signal emitted by the noise source
$E\{ \}$	expected value	N	number of coefficients in the adaptive filter
F_d	coefficient to scale the direct portion of the		W
	impulse responses A and B	N_1, N_2, N_B, N_S	variances of noise signal at microphone 1,
F_{σ}	coefficient to scale the reverberant portion of		microphone 2, sum of microphone signals,
	the impulse responses A and B		and output of beamformer, respectively
F _{sample}	sampling rate = $1/T_{\text{sample}}$, Hz	Р	cross-correlation vector
G_0	magnitude of the first coefficient of the im-	P_i	<i>i</i> th element of the cross-correlation vector <i>P</i>
	pulse responses A and B	$P_{d/r}$	direct-to-reverberant ratio of the noise signal
G_1	magnitude of the second coefficient of the		at location of the listener
	impulse responses A and B	$Q_{ m d/r}$	direct-to-reverberant ratio of the target sig-
G_{nR}	impulse response between noise source and		nal at location of the listener
	output signal of right microphone	r _c	critical distance, m
G_{nL}	impulse response between noise source and	R	autocorrelation matrix
	output signal of left microphone	S	signal emitted by the target source
G_{sR}	impulse response between target signal	$S(\vartheta)$	ratio between rms value of a white noise sig-
	source and output signal of right microphone		nal in free field and on the surface of a rigid
G _{sL}	impulse response between target signal		sphere
	source and output signal of left microphone	$S_{1}, S_{2}, S_{B}, S_{S}$	variances of target signal at microphone 1.
		1, 2, 5, 5	microphone 2, sum of microphone signals.
^{a)} Electronic mail: martin.kompis@insel.ch			and output of beamformer, respectively

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1	time constant for exponential decay of the
	filter coefficients modeling reverberation in
	impulse responses A and B, in multiples of
	the sampling period T_{sample}
T_r	reverberation time of room, s
T _{sample}	sampling period = $1/F_{\text{sample}}$, s
V	volume of room or enclosure, m ³
W	vector representing coefficients of the adap-
	tive filter
W^0	vector representing coefficients of the adap-
	tive filter in the adapted state
x	reference signal (difference of microphone
	signals)
Χ	vector of last N values of signal x
v	output of the adaptive filter
~	1 1

I. INTRODUCTION

Many users of cochlear implants and conventional hearing aids complain about insufficient intelligibility of speech in noisy situations, even if the performance of their aid is satisfactory in quiet environments (Kochkin, 1993). As many hearing impaired listeners need significantly higher signalto-noise ratios (SNR) for satisfactory communication than normal hearing listeners (Lurquin and Rafhay, 1996; Valente, 1998), numerous noise reduction methods for hearing aids and cochlear implants have been proposed (Lim and Oppenheim, 1979; Graupe et al., 1987; Soede et al., 1993; Bächler and Vonlanthen, 1995; Whitmal et al., 1996; Vanden Berghe and Wouters, 1998). Some of the most promising noise reduction schemes assume that target signals are emitted in front of the listener, while signals arriving from other directions are considered to be noise (Peterson et al., 1987; Soede et al., 1993; Bächler and Vonlanthen, 1995). Directional noise reduction methods have been shown to improve SNR and to be of practical use for the hard-ofhearing (Peterson et al., 1987; Greenberg and Zurek, 1992; Kompis and Dillier, 1994; Valente et al., 1995; Kochkin, 1996; Cochlear Inc., 1997; Gravel et al., 1999; Wouters et al., 1999). Several methods are known to achieve spatial directionality. Besides the use of directional microphones, the output signals of several (omnidirectional or directional) microphones can be postprocessed using either fixed or adaptive postprocessing (Soede et al., 1993; Kompis, 1998). In fixed postprocessing, all transfer functions between the microphone signals and the output are time independent. In adaptive postprocessing, the coefficients of at least one filter are continuously adjusted to optimize noise reduction in the given environment. In general, adaptive beamformers achieve higher noise reductions at the expense of higher computational loads and greater system complexity (De-Brunner and McKinney, 1995; Kates and Weiss, 1996; Kompis et al., 1999; Kompis et al., 2000).

While fixed beamformers have been theoretically analyzed and the achievable noise reduction can be predicted based on these theoretical considerations (Cox *et al.*, 1986; Stadler and Rabinowitz, 1993), predictions of the performance of adaptive systems are rare (Widrow *et al.*, 1975; Greek

- α_n azimuth of noise source
- α_s azimuth of target signal source
- Δ delay in target signal path between d' and d, in samples
- ϵ output signal of the adaptive beamformer
- σ_i^2 variance of the *i*th coefficient in filters A and B
- ϑ angle between point on surface of a rigid sphere and direction of incidence of plane wave
- γ_n index of directionality of the noise source
- γ_s index of directionality of the target signal source

Note: All parameters are dimensionless, unless otherwise noted

DeBrunner and McKinney, 1995). To date, they do not take into account the length of the adaptive filter and reverberation time of the environment, two factors which have been found to be of major importance (Peterson et al., 1987; Peterson et al., 1990; Kompis and Dillier, 1991; Greenberg and Zurek, 1992; Dillier et al., 1993). Most reports on adaptive beamformer applications provide experimental data using either speech recognition tests with normal hearing or hearing impaired listeners (Peterson et al., 1987; Kompis and Dillier, 1994; van Hoesel and Clark, 1995; Hamacher et al., 1996; Welker et al., 1997) or different measures related to signal-to-noise ratio improvement (Greenberg and Zurek, 1992; Greenberg et al., 1993; Dillier et al., 1993; Welker et al., 1997; Kates, 1997). It is difficult to compare the results of these reports because of the numerous differences in the experimental setting, such as reverberation time, directionality of sound sources or filter adaptation. The effect of each difference is hard to estimate because of the lack of a theoretical background or sufficient experimental data. In this report, the noise reduction that can be achieved by a two-microphone adaptive beamformer (Griffiths and Jim, 1982; Peterson et al., 1987) is analyzed. An approximate method to predict its noise reduction as a function of the design parameters of the beamformer and the acoustic parameters of the acoustic environment including the sound sources is derived. In Sec. II, the investigated adaptive beamformer is defined. In Sec. III, the assumptions for the theoretical analysis are discussed. Models of the impulse responses between the acoustic noise sources and the beamformer are presented in Sec. IV, and in Secs. V and VI, an approximation to predict the achievable improvement in signal-to-noise ratio is derived. Potential applications and limitations of the presented method to predict SNR improvements are discussed in Sec. VII. A short FORTRAN subroutine which performs the calculation to predict SNR improvement is included in the Appendix. Experimental verification of the predictions is provided in a companion paper (Kompis and Dillier, 2001).

II. THE ADAPTIVE BEAMFORMER

Figure 1 shows a schematic diagram of the twomicrophone adaptive beamformer (Griffiths and Jim, 1982;



FIG. 1. Schematic diagram of the adaptive beamformer in the acoustic environment used to predict SNR improvements.

Peterson *et al.*, 1987) considered in this research. Note that some researchers prefer the term *Griffiths–Jim beamformer* to describe the same system.

Two omnidirectional microphones are mounted close to the ears of a user. The sum and the difference of the two microphone signals is calculated first. As the target signal source is assumed to lie in front of the listener, the sum d'will contain predominantly target signal, while the difference signal x will contain mainly noise, as noise is assumed to arrive from other directions. A finite-impulse response structured adaptive filter W transforms x in such a way that it can serve as a model of the remaining noise in d. The resulting signal y can then be directly subtracted from d, yielding the output ϵ . The coefficients of the adaptive filter are updated by a least-mean-squares (LMS) algorithm (Widrow et al., 1975) which minimizes the total variance of the output signal. The LMS algorithm relies on the assumption that target and noise signals are uncorrelated. The delay in the target signal path between d' and d can be adjusted to optimize noise reduction. Typically, the length of the adaptive filter is chosen in the range of 10-50 ms, and delay is set to 25%-50% of the filter length (Peterson et al., 1987; Kompis and Dillier, 1991; Greenberg and Zurek, 1992; Dillier et al., 1993; Kompis and Dillier, 1994).

The adaptive beamformer minimizes the variance of any signal of which a—possibly linearly transformed—copy is present in the reference signal x. Due to reverberation and misalignment of the target signal source with respect to the microphones, in most practical situations a part of the target signal will be present in the reference signal x. To prevent target signal cancellation, several algorithms, which stop filter adaptation when a target signal is detected, have been proposed (Van Compernolle, 1990; Greenberg and Zurek, 1992; Kompis and Dillier, 1994; van Hoesel and Clark, 1995; Kompis *et al.*, 1997). Using one of these algorithms, filter adaptation is limited to time segments in which no target signal is present, e.g., the numerous short pauses that occur in the running speech of a target speaker.

III. MODEL ASSUMPTIONS

To predict the SNR improvement that can be achieved by the adaptive beamformer, a simplified model of the acoustic setting is assumed as follows (cf. the left-hand side of Fig. 1 for a graphic representation). A listener in a reverberant room faces a single target signal source. A second acoustic source, emitting the noise signal, is placed at an azimuth α_n from the listener, where α_n is large enough to give rise to a difference in the time of arrival of the noise signal between the two microphones of at least one sampling period T_{sample} . No movement of either the listener or the sound sources is allowed. The directionality of the acoustic sources is described by the index of directionality γ_n for the noise source and γ_s for the target signal source, defined as the ratio between the signal intensity emitted in the direction of the listener to the intensity of a hypothetical omnidirectional source with the same total acoustic output power (De-Brunner and McKinney, 1995). The head of the listener is modeled as a rigid sphere of 9.3 cm in radius, as proposed by Kuhn (1977) and used in an earlier study (Kompis and Dillier, 1993). Two omnidirectional microphones are mounted on the surface of the rigid sphere opposite each other, serving as inputs to the adaptive beamformer. The acoustic properties of the room are defined by any two of the three parameters volume V, reverberation time T_r , and critical distance r_c . Reverberation time is defined as the time required for the reverberant signal to decay by 60 dB. The critical distance is defined as the distance from an omnidirectional acoustic source at which the direct-to-reverberant ratio is 1. The relationship between these parameters can be approximated by

$$r_c \approx \sqrt{\frac{6\ln 10}{4\pi c}} \frac{V}{T_r},\tag{1}$$

where c is the sound speed (Zwicker and Zollner, 1984). For the calculations in the Appendix, a sound speed of c= 340 m/s is assumed. Both the noise and the target signal source are assumed to emit white noise, with the signals of the two sources being uncorrelated. The adaptive beamformer processing the two microphone signals is configured as shown in Fig. 1 and defined by its sampling rate F_{sample} , the number of coefficients N of the adaptive filter, and the number of samples Δ of delay in the target signal path between d' and d. A perfectly adapted filter is assumed, i.e., it is assumed that filter adaptation took place in the absence of the target signal and the coefficients of the adaptive filter have converged to their optimal state. The state of the adaptive filter is assumed to be frozen at the end of adaptation, so that only the noise signal, but not the target signal, has had an influence on the filter coefficients.

In principle, no restrictions are imposed by the model on the variances of either the noise or the target signal. However, in order to simplify calculations and without loss of generality, it is assumed that the variance of the noise signal n(k) equals 1, and room transfer functions are scaled in such a way as to let the variances of the noise signal equal 1 in both the sum signal d(k) and the difference signal x(k). Similarly, i.e., in order to simplify calculations and without loss of generality, the variance of the reverberant portion of the target signal at either microphone is assumed to be 1. Clearly, some of the above-mentioned assumptions are more limiting than others. The assumptions on the variances of the target and noise signals exclude situations without any reverberation. To generate a difference of at least one sampling period, at a sampling rate of, e.g., $F_{\text{sample}} = 10 \text{ kHz}$, the minimum azimuth of the noise source must be roughly 10°, which does not seriously limit general applicability. The model requires that the signals of both acoustic sources are white noise. Furthermore, effects of the frequency dependence of the acoustic diffraction by the head of the listener of the directionality of the sound sources are not taken into account. While this is clearly unrealistic in light of the predominantly low-frequency speech and noise sounds, which are to be expected as input signals in a hearing aid application, this assumption becomes more acceptable when considering that the most frequently used adaptation algorithm, the LMS algorithm (Widrow et al., 1975), minimizes total signal variance, i.e., the spectral components of a noise signal are reduced according to their relative power. Therefore, in numerous realizations of the adaptive beamformer, microphone signals are prewhitened by usually 6 dB per octave to account for the importance of the spectral components with respect to speech intelligibility (Peterson *et al.*, 1987; Dillier et al., 1993; Kompis and Dillier, 1994; Welker et al., 1997). Usually, changes introduced by these pre-emphasis filters are compensated by a de-emphasizing filter in the output path of the adaptive beamformer (Kompis, 1998). With these provisions, the spectra of the practically important speech signals actually being processed by the beamforming algorithm approach the white spectra of the model. Although it can be shown that broadband SNR improvement corresponds closely to an intelligibility-weighted measure of speech-tointerference ratio gain (Greenberg *et al.*, 1993) in numerous realistic experimental settings (Kompis and Dillier, 2001), the noninclusion of frequency dependence remains a limitation of the model. In the model of the listener, no pinnae or shoulders are accounted for. This simple model has been verified earlier and seems to be sufficient for a number of hearing aid applications (Kompis and Dillier, 1993). As there are several ways to mount hearing aid microphones with respect to the pinnae, and as the presented model does not generally take into account frequency dependence, the inclusion of pinnae or shoulder effects into the model does not seem to be justified. Again, however, the noninclusion of the alterations in the frequency spectra due to the head of the listener may be a limiting factor for a number of applications.

Although the two assumptions that (a) the filter has been adapted in the absence of the target signal and is (b) perfectly adapted cannot be expected to be met perfectly in real situations, these assumptions are reasonably realistic for many practical applications. Several target-signal detection/



FIG. 2. Relationship between the transfer functions G_{nR} , G_{nL} , G_{sR} , and G_{sL} and A, B, C, and D.

adaptation-inhibition algorithms have been proposed and used in experiments (Van Compernolle, 1990; Greenberg and Zurek, 1992; Kompis and Dillier, 1994; van Hoesel and Clark, 1995; Kompis *et al.*, 1997). Using one of these algorithms, it can be assumed that the target signal does not significantly influence filter adaptation and filter adaptation takes place in the presence of the noise signal only (Kompis *et al.*, 1997). At filter lengths of 10-50 ms, which are usually used for adaptive beamformers, short adaptation time constants on the order of magnitude of 0.1 s (Dillier *et al.*, 1993; Kompis and Dillier, 1994) can be combined with small convergence errors. Therefore, the coefficients of the adaptive filter can be reasonably expected to have converged, e.g., during the short pauses between the first words of an utterance of a target speaker.

IV. MODELING OF THE IMPULSE RESPONSES BETWEEN THE ACOUSTIC SOURCES AND THE MICROPHONES

The transfer functions between the two acoustic sources and the two microphones can be modeled as impulse responses G_{nR} , G_{nL} , G_{sR} , and G_{sL} , respectively. The first subscript (n or s) marks the source (noise or target signal), the second subscript (L or R) marks the left or right microphone. These impulse responses account for all effects of source directionality, room reverberation, and sound diffraction by the listener's head. For the analysis in Sec. V, it is convenient to convert these impulse responses into four slightly different impulse responses A, B, C, and D as follows:

$$A = G_{nR} + G_{nL} = (a_0, a_1, a_2, ...),$$

$$B = G_{nR} - G_{nL} = (b_0, b_1, b_2, ...),$$

$$C = G_{sR} + G_{sL},$$

$$D = G_{sR} - G_{sL}.$$
(2)

Using this definition, the calculation of the sum and difference of the microphone signals at the first stage of the adaptive beamformer is already included in A, B, C, and D, as shown schematically in Fig. 2.

While the impulse responses between the *target* sound source and the microphones do not influence filter adaptation and can therefore be handled in a simplified manner in Sec.



FIG. 3. Schematic representation of the model-transfer functions G_{nR} , G_{nL} , A, and B between noise source and the adaptive beamformer. The solid lines represent the directly incident portions of the noise signal, hatched areas represent the reverberant response.

VI, a more detailed model of the impulse responses between the *noise* source and the beamformer (i.e., G_{nR} , G_{nL} , A, and B) is required. These impulse responses are modeled by adding the direct response of the microphone which is closer to the noise source in coefficient 0, the direct response to the microphone farther away from it in coefficient 1, and the reverberation in coefficients 2 through ∞ , as depicted in Fig. 3. In general, the difference in the time of arrival between the two microphones will not be exactly one sampling period T_{sample} as modeled, but usually larger, e.g., four samples at a sampling rate of $F_{\text{sample}} = 10 \text{ kHz}$ and an azimuth of α_n =45° (differences smaller than T_{sample} are excluded by the model definitions in Sec. III). It was found that larger differences are negligible as long as the adaptive filter is much longer than the difference in the time of arrival. In most practical applications, filters are 10-100 times longer than the time-of-arrival difference of the noise sound and this prerequisite is met.

The size of the first two coefficients is a function of the angle of incidence of the direct, nonreverberated portion of the noise signal. The total rms value of a white noise signal at a point on the surface of a rigid sphere at an angle ϑ with respect to the angle of incidence and relative to the root-mean-square value of the same white noise in free field can be calculated from the formulas provided, e.g., by Schwarz (1943) or Morse (1983). Figure 4 shows the resulting function $S(\vartheta)$ for a rigid sphere with a radius of 9.3 cm for three different frequency bands of 0–2.5, 0–5, and 0–10 kHz, corresponding to sampling rates of 5, 10, and 20 kHz, if ideal nonaliasing filters are assumed. The differences between the three curves arise because of the more pronounced diffraction of the high frequency components of the signals.



FIG. 4. Sound pressure at the surface of a head-sized (r=9.3 cm) rigid sphere as a function of the angle of sound incidence ϑ . $S(\vartheta)$ represents rms values relative to free field, for white noise processed by three different low-pass filters.

Using $S(\vartheta)$, the first two coefficients of *A* and *B* can be written as

$$a_0 = b_0 = G_0 = S(\pi/2 - \alpha_n) F_d,$$

$$a_1 = -b_1 = G_1 = S(\pi/2 + \alpha_n) / F_d,$$
(3)

where F_d is a constant, the value of which will be determined shortly to account for the direct-to-reverberant ratio $P_{d/r}$ of the noise signal. All other coefficients, i.e., $a_i, b_i, i \ge 2$, representing the reverberant part of the room filter are modeled as a series of independent, normally distributed random variables, where

$$E\{a_i a_j\} = E\{b_i b_j\} = \begin{cases} 0, & i \neq j \\ \sigma_i^2, & i = j \end{cases}$$

$$E\{a_i b_j\} = 0$$
(4)

holds for all *i* and *j*. Note that for any given acoustic setting, *A* and *B* are linear impulse responses with fixed, well-defined and time-independent values a_i and b_i for all *i*. However, as the exact values of every a_i and b_i for the reverberant part $(i \ge 2)$ are neither known nor required for the following computation, only some relevant statistical properties of the coefficients are used. Nevertheless, the underlying impulse responses are time invariant and linear. The variance σ_i^2 decreases exponentially with the index *i* as the reverberant portion of the signal decays exponentially:

$$\sigma_i^2 = F_\sigma e^{-i/T},\tag{5}$$

where F_{σ} is another newly introduced coefficient to account for the correct direct-to-reverberant ratio and *T* is a time constant (dimensionless, in multiples of the sampling period T_{sample}).

To complete the model of the impulse responses A and B, the three newly introduced variables T, F_d , and F_σ must be calculated first. To derive the value of the dimensionless time constant T from the reverberation time T_r and the sampling period T_{sample} , the definition of the reverberation time (i.e., time required for the reverberant signal to decay by 60 dB) can be used:

$$e^{-T_r/TT_{\text{sample}}} = 10^{-60/10} \tag{6}$$

from which T can be calculated as

$$T = \frac{T_r F_{\text{sample}}}{6 \ln(10)}.$$
(7)

The direct-to-reverberant ratio $P_{d/r}$ of the noise signal at the location of the listener can be estimated as

$$P_{\rm d/r} = \left(\frac{r_c}{l_n}\right)^2 \gamma_n \,. \tag{8}$$

Using the two coefficients F_d and F_{σ} , it is possible to adjust the direct-to-reverberant ratio $P_{d/r}$ correctly,

$$\frac{G_0^2 + G_1^2}{\sum_{i=2}^{\infty} \sigma_i^2} = P_{d/r},$$
(9)

and at the same time guarantee that

$$\sum_{i=0}^{\infty} a_i^2 = \sum_{i=0}^{\infty} b_i^2 = G_0^2 + G_1^2 + \sum_{i=2}^{\infty} \sigma_1^2 = 1$$
(10)

as stated in Sec. III in order to keep calculations in the following sections as simple as possible. Using the identity

$$\sum_{i=M}^{N} e^{-i/T} = \frac{e^{-M/T} - e^{-(N+1)/T}}{1 - e^{-1/T}}$$
(11)

it can be found that

$$F_{d} = \sqrt{\frac{P_{d/r}}{(1 + P_{d/r})(S^{2}(\pi/2 - \alpha_{n}) + S^{2}(\pi/2 + \alpha_{n}))}}, \quad (12)$$

$$F_{\sigma} = \frac{1 - e^{-1/T}}{(1 + P_{d/r})e^{-2/T}}.$$
(13)

V. APPROXIMATE SOLUTION FOR THE AMOUNT OF NOISE SUPPRESSION BY THE ADAPTIVE FILTER

In this section, an approximate solution for the amount of noise reduction h provided by the adaptive filter, defined as

$$h = E\{d^2\} - E\{\epsilon^2\},\tag{14}$$

is derived. The noise reduction h for an ideally adapted filter can be calculated analytically if the delayed sum signal d(k)and the reference signal x(k) are known. The derivation of the corresponding equations can be found in standard textbooks (e.g., Widrow and Stearns, 1985) on adaptive filters and is not repeated here. To calculate the *approximate* noise reduction for the problem of the adaptive beamformer in a reverberant room, the following definitions are needed. Let Xbe a vector of the last N samples in the reference signal x, where N is the number of coefficients in the adaptive filter. Then an autocorrelation matrix R can be defined as

$$R = E\{X \cdot X^T\},\tag{15}$$

where the superscript *T* stands for transposition and $E\{ \}$ denotes the expected value over time. Similarly, let the cross-correlation vector *P* be

$$P = E\{X \cdot d\} = \begin{bmatrix} P_0 \\ P_1 \\ \vdots \\ P_{N-1} \end{bmatrix}.$$
 (16)

Using these definitions, the vector W^0 containing the *N* filter coefficients of the ideally adapted filter for which the variance of the output signal $E\{\epsilon^2(k)\}$ becomes minimal can be written as

$$W^0 = R^{-1} P. (17)$$

The noise reduction h can then be expressed as

$$h = E\{d^2\} - E\{\epsilon^2\} = W^{0T}P = P^T R^{-1}P.$$
 (18)

For the investigated problem, signals x and d are not known. However, as the source signal n is known to be white noise signal with variance 1, the samples of n are known to be statistically independent. Using the coefficients a_i and b_i of the impulse responses A and B, the elements of the cross-correlation vector P can then be written as

$$P_i = \sum_{k=\max(0,\Delta-i)}^{\infty} a_k \cdot b_{k-i+\Delta}.$$
 (19)

As long as the samples of the noise signal remain statistically independent in the reference signal x(k), i.e., *after* modification by the impulse response *B*, the autocorrelation matrix *R* can be approximated by the identity matrix *I*,

$$R \approx I.$$
 (20)

However, this approximation is reasonably accurate only for low direct-to-reverberant ratios $P_{d/r}$ of the noise signal, where the statistically independent coefficients of the reverberant response dominate the impulse response B. At high direct-to-reverberant ratios of the noise signal, B and therefore x(k) are dominated by the directly incident noise portions and the assumption of statistically independent samples x(k) is violated. It can be shown (Kompis and Dillier, 2001) that the given approximation is reasonably accurate for direct-to-reverberant ratios of the noise signal $P_{d/r}$ < +3 dB. Using this approximation and Eqs. (18)+(19), the noise reduction h could be calculated if all model coefficients a_i and b_i were explicitly known. Except for a_0 , a_1 , b_0 , and b_1 however, only the expected value, which is zero, and the expected variance, which is σ_i^2 , are known. Therefore h cannot be calculated, but its expected value $E\{h\}$ can be approximated by

$$E\{h\} = E\{P^{T}R^{-1}P\} \approx \sum_{i=0}^{N-1} E\{P_{i}^{2}\}.$$
(21)

There is a meaningful interpretation of this equation. To simplify the discussion, let the delay in the target signal path Δ equal zero for this paragraph only. Equation (21) shows that in order to calculate the expected value of the noise reduction *h*, *N* positive values $E\{P_i^2\}$ are summed, thus increasing the noise reduction *h* with the length of the adaptive filter [P_i can never equal zero because of Eq. (19)]. Using the program in the Appendix it can even be shown that $E\{h\}$ approaches 1 (i.e., perfect noise cancellation) for any reverberation time with increasing filter lengths *N*, as long as the

directly incident portion of the sound remains negligible. From the schematic representation of the impulse responses A and B and the definition of P_i in Eq. (19) it can be seen that in environments with short reverberation times T_r , only the first few coefficients a_i and b_i will contribute significantly to P_i^2 , and P_i^2 will therefore only contribute significantly to $E\{h\}$ for small values of the index *i*. Calculating the contribution of the terms with large values of the index i is equivalent to shifting the impulse responses A and B significantly with respect to each other before multiplying and summing the corresponding coefficients in Eq. (19). Therefore, in situations with short reverberation times, after the first few terms in Eq. (21), $E\{h\}$ will increase only very slowly with N, meaning that already short adaptive filters can significantly reduce noise. For long reverberation times, the reverberant tails in Fig. 3 become long as well, but the first few coefficients a_i and b_i are smaller than for short reverberation times because of Eq. (10). This means that the contribution of the first few of the N filter coefficients of the adaptive beamformer are smaller than at short reverberation times, but the increase in noise reduction of the N+1st coefficient of the adaptive filter is larger for large N and longer filters will be needed to reach the same amount of noise reduction. At high direct-to-reverberant ratios $P_{d/r}$ of the noise source, the first two coefficients in A and B (a_0, a_1, a_1, a_1) b_0 , and b_1) representing the direct response are large, and the effect is similar to that of shortening reverberation time. Because of the approximation [Eq. (20)] used, Eq. (21) is only valid if the $P_{d/r}$ is small, i.e., less than approximately +3 dB (Kompis and Dillier, 2001). This is a new assumption which was not discussed in Sec. III and which limits the range of applicability of the given analysis. As a consequence, achievable gains in signal-no-noise ratio will be underestimated for situations with high direct-to-reverberant ratios of the noise source. Consequences will be discussed in Sec. VII.

To estimate $E\{h\}$, each of the N terms of the sum in Eq. (23) must be calculated first. Each term is itself a sum, which can be conveniently split into three terms as follows:

$$E\{P_i^2\} = E\left\{\left(\sum_{k=\max(0,\Delta-i)}^{\infty} a_k \cdot b_{k-i+\Delta}\right)^2\right\}$$
$$= E\left\{\left(\sum_{k=\max(0,\Delta-i)}^{\infty} a_k \cdot b_{k-i+\Delta}\bigg|_{\substack{k<2\wedge\\k-i+\Delta<2}}\right)^2\right\}$$
$$+ E\left\{\left(\sum_{k=\max(0,\Delta-i)}^{\infty} a_k \cdot b_{k-i+\Delta}\bigg|_{\substack{k<2\oplus\\k-i+\Delta<2}}\right)^2\right\}$$
$$+ E\left\{\left(\sum_{k=\max(0,\Delta-i)}^{\infty} a_k \cdot b_{k-i+\Delta}\bigg|_{\substack{k\geq2\wedge\\k-i+\Delta\geq2}}\right)^2\right\}$$
$$= \varphi_{dd}(i) + \varphi_{dr}(i) + \varphi_{rr}(i). \tag{22}$$

The three portions cover the terms concerning the directly incident portion of the noise only (φ_{dd}), the terms concerning the reverberant terms only (φ_{rr}), and the mixed terms (φ_{dr}). As a_0 , a_1 , b_0 , and b_1 are explicitly known from Eq. (4), φ_{dd} can be directly calculated as follows:

$$\varphi_{\rm dd}(i) = \begin{cases} (G_0^2 - G_1^2)^2, & |i - \Delta| = 0\\ (G_0 \cdot G_1)^2, & |i - \Delta| = 1\\ 0, & |i - \Delta| \ge 2. \end{cases}$$
(23)

For the mixed term φ_{dr} the properties

$$E\{(\xi_1 + \xi_2)^2\} = E\{\xi_1^2\} + E\{\xi_2^2\},\$$

$$E\{(\xi_1 \cdot \xi_2)^2\} = E\{\xi_1^2\} \cdot E\{\xi_2^2\},\$$
(24)

of any two independent random variables ξ_1 and ξ_2 can be used, as all a_i and b_i are independent of each other for $i \ge 2$ and independent from G_0 and G_1 . The result yields

$$\varphi_{\rm dr}(i) = \sum_{k=\max(0,\Delta-i)}^{\infty} E\{a_k^2\} \cdot E\{b_{k-i+\Delta}^2\} \bigg|_{\substack{k<2\oplus\\k-i+\Delta<2}} \\ = \begin{cases} 0, \quad |i-\Delta|=0\\ G_1^2 \cdot \sigma_{|i-\Delta|+1}^2, \quad |i-\Delta|=1\\ G_0^2 \cdot \sigma_{|i-\Delta|}^2 + G_1^2 \cdot \sigma_{|i-\Delta|+1}^2, \quad |i-\Delta| \ge 2. \end{cases}$$
(25)

Similarly, using Eq. (11), the reverberant term $\varphi_{\rm rr}$ can be calculated as

$$\varphi_{\rm rr}(i) = \sum_{k=\max(0,\Delta-i)}^{\infty} E\{a_k^2\} \cdot E\{b_{k-i+\Delta}^2\} \bigg|_{\substack{k \ge 2 \land \\ k-i+\Delta \ge 2}}$$
$$= \sum_{k=2}^{\infty} \sigma_k^2 \cdot \sigma_{k+|\Delta-i|}^2$$
$$= \frac{F_{\sigma}^2 \exp\left(-\frac{4+|\Delta-i|}{T}\right)}{1-e^{-2/T}}.$$
(26)

By substituting Eqs. (23), (25), and (26) into Eq. (22), using Eq. (21) an approximation for $E\{h\}$ can now be calculated.

VI. IMPROVEMENT OF THE SIGNAL-TO-NOISE RATIO

To estimate SNR improvement, the level of the target signal and of the noise signal will be compared at the following four different points of the signal processing chain (cf. Fig. 1) of the adaptive beamformer: (i) at the microphone with the less favorable SNR lying closer to the noise source (index 1), (ii) at the microphone with the more favorable SNR lying farther away from the noise source (index 2), (iii) after summation of both microphone signals, i.e., signal d' in Fig. 1 (index S), and (iv) at the output of the adaptive beamformer, i.e., signal ϵ in Fig. 1 (index B). By calculating the SNRs in those four signals, the SNR improvement of the adaptive beamformer can be related to either microphone signal or to the SNR gain of a simple fixed two-microphone beamformer (Kompis and Diller, 1994), in which both microphone signals are summed.

To calculate the level of the target signal in these four signals, the direct-to-reverberant ratio of the target signal $Q_{d/r}$ at the location of the listener can be estimated—in analogy to Eq. (8)—as

$$Q_{\rm d/r} = \left(\frac{r_c}{l_s}\right)^2 \cdot \gamma_s \,. \tag{27}$$

As discussed in Sec. V, reverberation must be present for the approximation (20) to be valid. Without loss of generality, the variance of the reverberant portion of the target signal can therefore be set to 1, and the total variance (i.e., including the direct *and* reverberant portions) of the target signal in the two microphones becomes

$$S_1 = S_2 = 1 + Q_{\rm d/r}.$$
 (28)

By adding both microphone signals, which corresponds to the signal processing of a part of the front end of the adaptive beamformer, the variance of the (uncorrelated) reverberant portion is doubled, while, assuming perfect alignment of the target source, the *amplitude* of the direct portion of the sound is doubled, and therefore its variance is multiplied by a factor of 4. However, this is only true for perfect alignment of the target signal source with respect to the microphones. In a realistic setting, e.g., for head-sized spacing between the microphones and for a sampling rate of, e.g., $F_{\text{sample}} = 10 \text{ kHz}$, this is valid for azimuths of the target signal source $\alpha_s = -3^\circ \dots + 3^\circ$. If the misalignment gives rise to a time difference of more than approximately T_{sample} , which in the above-mentioned example occurs at $\alpha_s > 10^\circ$, uncorrelated samples of the white noise signal will add up and the variance of the direct portion of the signal is only doubled. To account for this effect, an alignment factor A is introduced, which can be assessed experimentally in anechoic environments and will, for white noise, yield values in the range of 4 (perfect alignment) down to approximately 2 (no alignment). The variance of the target signal portion in the sum d' can thereby be written as

$$S_S = 2 + A \cdot Q_{d/r}. \tag{29}$$

Similarly, the variance of the target signal in the reference path x becomes

$$S_D = 2 + (4 - A) \cdot Q_{d/r}.$$
 (30)

As, according to the model assumptions, noise and target signal are uncorrelated and as the filter W was adapted in the absence of the target signal, the variance of the target signal portion in the reference signal x will increase by the factor of $W^{0T}W^0$ at the output of the adaptive filter (signal y). Using Eq. (17) and approximations (20) and (21), this factor can be shown to be equal to $E\{h\}$. The variance of the target signal at the output ϵ of the adaptive beamformer can now be written as the sum of the variances of the filtered reference signal y and the delayed sum signal d,

$$S_B = S_S + E\{h\} \cdot S_D \,. \tag{31}$$

So much for the target signal. As to the signal of the noise source, its variance in the sum signal d' can be set to 1 without loss of generality:

$$N_{\rm S} = 1.$$
 (32)

The variance of the noise signal at the output of the beamformer can then be written as

$$N_B = 1 - E\{h\}.$$
 (33)

The variance of the reverberant portion of the noise in the microphone signals is on average $\frac{1}{2}$ of that of the sum signal, the direct portion of the noise is not changed, thus

$$N_1 = \frac{1}{2} \cdot \frac{1}{P_{d/r} + 1} + G_0^2, \quad N_2 = \frac{1}{2} \cdot \frac{1}{P_{d/r} + 1} + G_1^2.$$
(34)

Now the improvement in SNR at the output of the adaptive beamformer, when compared to the SNR the microphone with the less favorable SNR (V_1), to the microphone with the more favorable SNR (V_2), or when compared to the two microphone fixed beamformer (V_S) can be calculated as follows:

$$V_{1} = 10 \log_{10} \frac{S_{B} \cdot N_{1}}{N_{B} \cdot S_{1}},$$

$$V_{2} = 10 \log_{10} \frac{S_{B} \cdot N_{2}}{N_{B} \cdot S_{2}},$$

$$V_{S} = 10 \log_{10} \frac{S_{B} \cdot N_{S}}{N_{B} \cdot S_{S}}.$$
(35)

The FORTRAN subroutine provided in the Appendix performs all computations necessary to determine all three SNR improvements in Eq. (35).

VII. DISCUSSION

The presented procedure used to estimate the SNR improvement of an adaptive beamformer in the given model setting is based on a number of assumptions and approximations. Its applications are therefore limited. A set of underlying assumptions have been listed and discussed in Sec. III. One additional limitation concerning the range of validity of the predictions is not listed in Sec. III, as it is not a consequence of the underlying model but rather of the approximation used in Eq. (20). For this approximation to be applicable, the direct-to-reverberant ratio $P_{d/r}$ of the noise source must be small, as stated in Sec. V. This limits the predictions to situations with at least a small level of reverberation. It can be shown experimentally (Kompis and Dillier, 2001) that, for realistic sets of parameter values, it is sufficient for $P_{d/r}$ to be below approximately +3 dB for reasonably accurate predictions. For higher $P_{d/r}$, SNR improvement will be systematically underestimated. However, for many applications, this is not a serious limitation. As the model is limited to low direct-to-reverberant ratios of the *noise* source only, predictions for high direct-to-reverberant ratios of the target signal source are not affected by this limitation. Although as a side effect of the precedence effect it may not always be easy to appreciate the amount of reverberation subjectively, in many acoustic settings in rooms with realistic amounts of reverberation direct-to-reverberant ratios are below +3 dB even at distances well below 1 m (Kompis and Dillier, 2001), and users of the system will probably tend to keep away from disturbing noise sources, thus further decreasing direct-to-reverberant ratio. Mainly in anechoic environments, however, where the adaptive beamformer is known for its excellent performance (Peterson *et al.*, 1987), the presented method does not adequately predict SNR improvement.

For hearing aid applications, the primary goal is improved speech intelligibility and not improved SNR, as predicted by the presented method. Because some frequency bands contribute more to speech intelligibility than others, SNR improvement may correlate poorly with improvement in speech recognition, if substantial differences between SNR improvements in different frequency bands exist. However, it can be shown that in the present context, SNR and intelligibility-weighted gain (Greenberg *et al.*, 1993) agree reasonably for a wide range of relevant experimental conditions (Kompis and Dillier, 2001).

The validation of the predicted SNR improvements is of major importance. Validation of the prediction procedure by comparisons to published experimental data is complicated by several factors. Comparisons are limited to experiments which meet or at least approach the model assumptions listed in Sec. III. Comparisons are not possible if different numbers or arrangements of microphones or several noise sources are used (e.g., Peterson et al., 1990; Greenberg and Zurek, 1992). As the proposed prediction method is limited to reverberant conditions, comparisons with experiments in anechoic environments (Peterson et al., 1987; Peterson et al., 1990; Greenberg and Zurek, 1992) are not meaningful. Some of the results reported in the literature list the improvement in terms of speech recognition scores rather than SNR improvement, and in some instances it is not possible to extract the latter information from these data (Kompis and Dillier, 1994; van Hoesel and Clark, 1995). In some reports (van Hoesel and Clark, 1995; Hamacher et al., 1996), no data on the directionality of the sound sources are given. Directionality of the sound sources are required input parameters to calculate the predicted SNR improvement using the presented method. For these reasons, a series of 92 experiments using the adaptive beamformer was performed and experimental results were compared to the predicted SNR improvements. These data are reported separately (Kompis and Dillier, 2001).

Despite some limitations, the presented prediction method offers several advantages over actual experiments in real or simulated environments. Results for a wide range of acoustic settings can be obtained in a fraction of the time required for actual experiments. Results are substantially less prone to errors and problems in the experimental setting such as programming errors, inadvertently wrong entry of simulation data, wiring or microphone problems, etc. Furthermore, predictions are not influenced by technical limitations of experimental settings such as limited resolution of analog-todigital converters, nonideal adaptation of the adaptive filter, effects of electrical or acoustic noise, etc. Therefore, the predictions offer a unique method to differentiate between implementational and/or experimental limitations and limitations of the adaptive beamforming method per se. Even if the prediction method is not used, it may be helpful for experiments by providing a list of parameters which have to be controlled in every experiment.

The presented prediction method cannot be expected to

replace experiments completely, but experiments and predictions can complement each other favorably. One potential application of the presented algorithm is to enable a validation of experimental data, e.g., if experimental results are either unexpectedly favorable or unexpectedly poor. If the predictions are sufficiently verified experimentally, many time-consuming experiments can be even omitted completely in the early stages of the development of a practical adaptive beamforming noise reduction system.

Probably the most interesting application is the study of the complex behavior of the adaptive beamformer in a wide variety of acoustic situations within a reasonable time span. A first effort in this direction is presented in a companion paper (Kompis and Dillier, 2001).

Because of the numerous underlying assumptions and the approximation used, there is considerable room for improvement for the presented prediction algorithm. Extension to situations with higher $P_{d/r}$, to frequency-dependent predictions of the SNR improvement, or extensions to cases using other numbers or arrangements of microphones (Peterson *et al.*, 1990; Greenberg and Zurek, 1992; Kates and Weiss, 1996) or directional microphones (Kompis and Dillier, 1994; DeBrunner and McKinney, 1995) might prove to be very useful.

To perform the relatively complex calculations to predict SNR improvements, a FORTRAN subroutine is provided in the Appendix. FORTRAN was chosen as it is still one of the most widely used programming languages among scientists and engineers (Kornbluh, 1999) and its code can be easily translated to other programming languages.

Despite the above-discussed drawbacks and limitations, the presented method to predict the SNR improvement of adaptive beamformer may be a useful tool in the design and further development of adaptive multimicrophone noise reduction systems for conventional hearing aids and cochlear implants. With its unique possibility to preliminarily evaluate different adaptive beamformers in a wide range of acoustic settings, it may help to point to new directions in research by showing where inherent limitations of the current adaptive beamformer design need to be overcome by innovative concepts.

VIII. SUMMARY

A method to predict the SNR improvement of a twomicrophone adaptive beamformer in a reverberant environment has been presented. Predictions are limited to static situations with one noise and one target signal source and perfect adaptation of the adaptive filter is assumed. A FORTRAN subroutine to perform the necessary calculations has been provided. A systematic validation study of the predictions is provided in a separate text (Kompis and Dillier, 2001).

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APPENDIX: FORTRAN SUBROUTINE

```
SUBROUTINE BEAMF (V, Tr, Fls, GammaS, A, Fln,
          GammaN, AlphaN, Fsampl, N, ID, V1, V2, Vs)
     1
C Input:
С
  V Volume of room (m3)
С
  Tr
          Reverberation time Tr (s)
С
   Fls
          Distance listener - target signal
С
           source (m)
С
   GammaS Index of directionality of target
С
           signal source
С
          Alignment factor for target signal
   А
С
           source (range 2 to 4)
С
   Fln
          Distance listener-noise source (m)
С
   GammaN Index of directionality of target
С
           signal source
С
   AlphaN Azimuth of noise source (degrees)
С
   Fsampl Sampling rate (Hz)
С
          Number of coefficients in the
   Ν
С
           adaptive filter
С
   ID
          Delay in target signal path (in
С
           multiples of 1/Fsampl)
С
  Output:
С
   V1
          SNR-improvement vs.
С
           signal of microphone 1 (dB)
С
   V2
          SNR-improvement vs.
С
           signal of microphone 2 (dB)
C
  Vs
          SNR-improvement vs.
С
           sum of microphone signals (dB)
C Array for function S(theta) for 3 sampling
С
   rates Fsampl=5,10, and 20kHz
      DIMENSION S(3,11)
      DATA(S(1,I),I=1,4)/1.59,1.57,1.52,1.42/
      DATA(S(1,I), I=5,8)/1.29,1.13,0.97,0.84/
      DATA(S(1,I),I=9,11)/0.76,0.93,1.10/
      DATA(S(2,I),I=1,4)/1.78,1.75,1.69,1.59/
      DATA(S(2,I), I=5,8)/1.42,1.21,0.98,0.79/
      DATA(S(2,I),I=9,11)/0.68,0.72,1.09/
      DATA(S(3,I),I=1,4)/1.85,1.86,1.81,1.73/
      DATA(S(3,I), I=5,8)/1.53,1.26,0.96,0.71/
      DATA(S(3,I),I=9,11)/0.57,0.58,1.06/
      rc=0.057*SQRT(V/Tr) ! critical distance
      Pdr=SQR(rc/Fln)*GammaN
      Qdr=SQR(rc/Fls)*GammaS
                            ! 13.815=6*ln 10
      T=Tr*Fsampl/13.815
      IFS=2
      IF(Fsampl.LT. 7500.) IFS=1
      IF(Fsampl.GT.15000.) IFS=3
C limit AlphaN to values of 0...90 degrees
C because of symmetry
      AlphaN=ABS(AlphaN)
      IF(AlphaN.GT.90.) AlphaN=180.-AlphaN
C S(theta) for position of each microphone
    (ST0,ST1) by interpolation
C
      RINDEX=(90.-AlphaN)/18.+1.
```

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```
INDEX=INT (RINDEX)
      RINDEX=RINDEX-INDEX
      ST0=S(IFS, INDEX) * (1.-RINDEX) +
     1 S(IFS, INDEX+1) *RINDEX
      RINDEX=AlphaN/18.+6.
      INDEX=INT (RINDEX)
      RINDEX=RINDEX-INDEX
      ST1=S(IFS, INDEX) *
       (1.-RINDEX) +S(IFS, INDEX+1) *RINDEX
     1
      Fd=SQRT(Pdr/((1.+Pdr)*
     1 (SQR(ST0)+SQR(ST1))))
      Fsigma=(1.-EXP(-1./T))/
     1 ((1+Pdr) * EXP(-2./T))
      G0=Fd*ST0
      G1=Fd*ST1
      h=0.
      DO 100 I=0,N-1
      PHIdd=0.
      IF(IABS(I-ID).EQ.1) PHIdd=SQR(G0*G1)
      IF(I.EQ.ID) PHIdd=SQR(G0*G0-G1*G1)
      PHIdr=0
      IF(IABS(I-ID).EQ.1) PHIdr=G1*G1*
     1 SIGMA2 (IABS (I-ID) +1, Fsigma, T)
     IF(IABS(I-ID).GE.2) PHIdr =
     1 G0*G0*SIGMA2(IABS(I-ID)
                                  ,Fsigma,T)+
     1 G1*G1*SIGMA2(IABS(I-ID)+1,Fsigma,T)
      PHIrr=SQR(Fsigma)*
     1 EXP((-4-IABS(I-ID))/T)/(1.-EXP(-2./T))
      P2=PHIdd+PHIdr+PHIrr
      h=h+P2
100
      CONTINUE
      S1=1.+Qdr
      S2=S1
      Ss=2.+A*Qdr
      SD=2.+(4.-A)*Odr
      SB=Ss+h*SD
      FNs=1.
      FNB=1.-h
      FN1=0.5/(1.+Pdr)+G0*G0
      FN2=0.5/(1.+Pdr)+G1*G1
      V1=10.*ALOG10((SB*FN1)/(FNB*S1))
      V2=10.*ALOG10((SB*FN2)/(FNB*S2))
      Vs=10.*ALOG10((SB*FNs)/(FNB*Ss))
      RETURN
      END
      FUNCTION SIGMA2(I, Fsigma, T)
      SIGMA2=Fsigma*EXP(-I/T)
      RETURN
      END
      FUNCTION SQR(X)
      SOR=X*X
      RETURN
      END
```

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